On Second Thought? A New Perspective on Secondary Moments

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ON SECOND THOUGHT? A NEW PERSPECTIVE ON SECONDARY MOMENTS

BY JONATHAN HIRSCH

One of the rites of passage of any engineer learning how to design post-tensioned concrete is to understand the meaning and usage of secondary moments. There is a vast amount of information available on calculating and applying secondary moments. Many engineers follow a traditional approach for designing post-tensioned members using secondary moments, which can result in code-compliant and safe designs. This document will provide a deeper understanding of where this approach came from, why and when it works, and demonstrate that it is only one approach for designing post-tensioned members.

INTRODUCTION

Post-tensioning is one of the strongest and most resilient ways to reinforce concrete. However, the calculation of sectional strength in post-tensioned members is slightly more complicated than that of a corresponding reinforced concrete member due to the presence of so-called secondary effects. One of the first things designers are taught is that post-tensioned members require the consideration of secondary moments, often without a detailed explanation of why and when they are necessary.

ACI 318-19[1] provides definitions for secondary effects that can become confusing to apply in some common situations. Situations that require caution include the presence of rigid columns or walls, which can restrain shortening, or in continuum structure analysis such as finite element analysis of floors. The use of secondary effects will be clarified so that the reader can understand why they are used and how they can be included appropriately in design.

Note: The sign conventions used throughout this document are that positive moments induce tension on the bottom of the member. Axial forces and stresses are tension positive, compression negative.

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ENSURING FLEXURAL STRENGTH CAPACITY OF CONCRETE MEMBERS

The flexural strength capacity of concrete members must be calculated to ensure that they have adequate strength to resist the applied demand loads. This normally occurs by selecting appropriate cross sections in the floor, calculating the demand forces on those cross sections caused by externally applied loads, and calculating the capacity of the cross sections using methods permitted by the building code or standard.

Flexural and axial strength in concrete cross sections is normally satisfied by ensuring that the internal material stresses can resist the externally applied loads. The analysis for shallow members is carried out assuming plane sections remain plane and using strain compatibility to calculate the material strains and stresses. In non-prestressed reinforced concrete cross sections, the lever arm between the concrete and reinforcing bar remains approximately constant, while, with increasing load, the stresses in both the steel and concrete increase proportionally (Fig. 1(a) to (c)). The ultimate state is reached when the strain in the concrete reaches a limit strain that precludes crushing of the compression side, with the reinforcing bar normally yielding in tension (Fig. 1(d)). The distance between the resultant concrete compression force and the resultant reinforcing bar tension force at the ultimate state is the lever arm, z. For reinforced concrete, the nominal flexural capacity with no external axial load is then calculated as

Note from the editors: This paper presents a different approach to understanding the meaning and use of secondary moments. The process described in no way intends to negate code-required calculations or loading considerations—it provides a different calculation method resulting in the same end result. This method is suitable for use with computer methods, provides an alternate means to understanding secondary moments, and provides an alternate approach for designing post-tensioned members.

$$M_{n0} = A_s f_y z \qquad (1)$$

where A_s is the area of non-prestressed reinforcement;

f_y is the yield stress of non-prestressed reinforcement; and

z is the depth between the reinforcement and resultant concrete compression force.

The flexural strength behavior of post-tensioned concrete members is quite different from reinforced concrete. As the tendons are stressed, beneficial compression stresses are introduced into the concrete. This optimizes the material behavior because concrete is strong in compression but relatively weak in tension. Stressing of the tendons induces an initial state of stress in the concrete (Fig. 2(a)) that includes a flexural component and a uniform precompression component. As external gravity loads are increased, the concrete compression stress resultant shifts away from the tendon tension force, thereby increasing the lever arm between the two. At the point where the externally applied gravity moment exactly counteracts the flexural prestress component, the compression stress in the concrete is uniform (Fig. 2(b)). As more load is applied, additional strength is derived from the shifting of the compression stress resultant from the centroid of the section to the final compression block resultant location (Fig. 2(c) and (d)). This behavior is characterized by the tension and compression forces remaining approximately constant, while the lever arm increases with applied load.

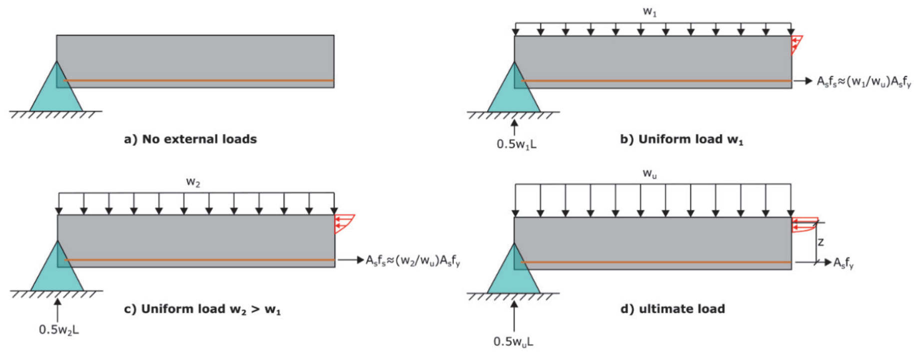


Fig. 1—Reinforced concrete strength behavior.

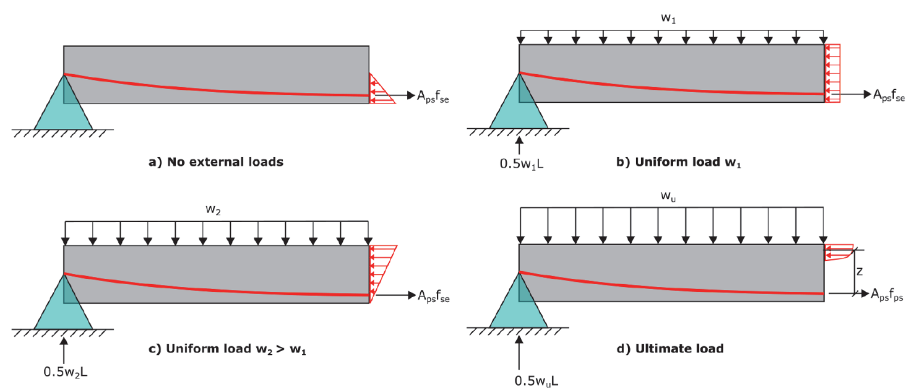


Fig. 2—Post-tensioned concrete strength behavior.

When the tendon strains get large as the ultimate load is approached, the tendon stress also increases from the effective stress f_{se} to the ultimate tendon stress f_{ps} (Fig. 2(d)). These stress increases are based upon strain compatibility for bonded tendons and are approximately proportional to the strain increase for unbonded tendons. ACI 318-19[1] provides equations to predict the ultimate tendon stress, f_{ps}.

When calculating the flexural strength of post-tensioned cross sections, the entire tendon stress is normally included in the strain compatibility calculations. The equation that is used to calculate the nominal flexural capacity with no external axial loads using the traditional approach is often taken as

$$M_{n0}' = A_{ps} f_{ps} z \qquad (2)$$

where A_{ps} is the area of prestressed reinforcement; and

f_{ps} is the ultimate stress of the prestressing strand.

What are secondary moments and why are they necessary?

Equation (2) looks very similar to Eq. (1) and implies that the presence of a stressed strand contributes to the strength of post-tensioned sections in the same way that the presence of reinforcing bar contributes to the strength of reinforced concrete sections. However, this does not represent the actual capacity behavior. Most of the strength of post-tensioned sections is derived from the beneficial prestrains (initial compression strains) in the concrete generated by stressing the strands and is not directly related to the presence of a stressed strand. This can be illustrated using a simple, yet contrived example.

Consider a simply supported beam with a rectangular cross section that is pre-cracked through the entire depth at midspan (Example 1). The pre-cracked condition mimics the design assumption that ignores the tensile strength of concrete in cross section calculations (ACI 318-19, Section 22.2.2.2). A strand can be introduced in this cross section and anchored to an exterior fixed structure. For simplicity, assume that the strand is in a frictionless sheathing, and as such, the strand stress does not increase significantly with increasing load. For this example, the catenary action of the strand is also ignored as this behavior is generally not included in flexural strength calculations.

The free-body diagram in Example 1(b) demonstrates that there is no way to generate any compression in the concrete, and hence no way to generate any flexural capacity. This simple example illustrates that the presence of a stressed strand in a concrete cross section does not directly contribute to the flexural strength. Using Eq. (2), which uses the entire strand stress, therefore requires additional modifications to maintain the equilibrium of the demand versus capacity comparison. These modifications, as may be guessed, are referred to as secondary effects.

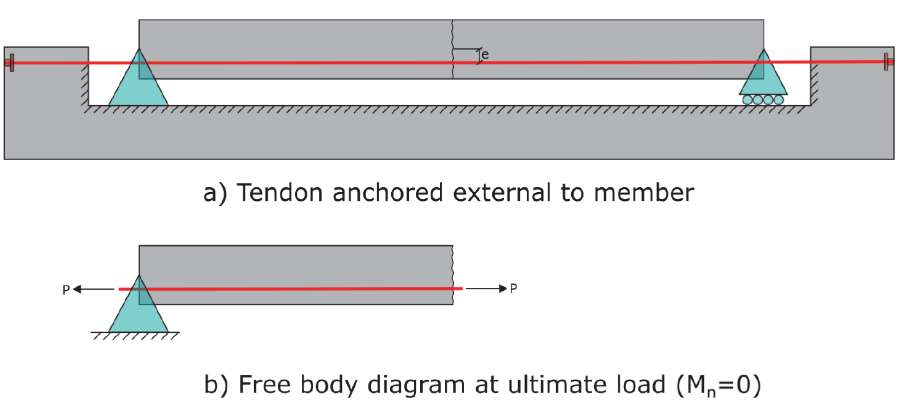
POST-TENSIONED FLEXURAL STRENGTH CAPACITY—ACI SECONDARY FORCE APPROACH

ACI 318-19 Section 22.2.1.3 prescribes that the effective stress in the strand f_{se} be included in the cross section strain compatibility calculations in addition to the increase in strand stress from f_{se} to f_{ps} as the behavior approaches ultimate. Secondary effects as defined by ACI are the set of forces induced by the resistance caused by the restraining effect of the supports during the stressing process. They are applied to the demand force, and not the resistance. Because Eq. (2) is only valid with the inclusion of secondary effects, it is considered a pseudo-capacity and denoted by the term M_{n0}' to distinguish it from M_{n0} calculated by Eq. (1), which is considered a true capacity that can be compared directly with the demand forces caused by externally applied loads, M_u. The idea behind the ACI definition is that the application of prestressing to the member can induce support reactions, which should be treated as additional external loads on the member. This definition of secondary effects, while perhaps appropriate for simple isolated frame structures, can be confusing and in many cases incorrect to apply to more complicated structures that are nowadays common, like finite element analysis of entire floors. The reader is referred to a previous publication by Bommer[2] for a more thorough discussion of this topic.

Application of the ACI definitions to Example 1 can become very confusing. The member being analyzed does not have any support reactions caused by prestressing, so the conclusion could be made that there are no secondary effects by the ACI definitions. However, this would result in the calculation of significant flexural capacity on this cross section using Eq. (2), which is obviously incorrect. This example demonstrates the potential confusion caused by using the ACI definitions in some cases, and for this reason, it is recommended that the ACI definition for secondary moments be modified.

POST-TENSIONED FLEXURAL STRENGTH CAPACITY—NEW EQUILIBRIUM SOLUTION

Figure 2 illustrates that a significant portion of the strength of post-tensioned cross sections is a function of the internal compression strains and stresses generated from stressing the strands. The strength contribution from Fig. 2(a) to (b) is created from the counteraction of the beneficial flexural effects caused by stressing the tendons. These beneficial flexural effects could be generated from drape on the strands or by eccentric anchorage locations. There is an additional strength contribution from Fig. 2(b) to (d) as the uniform precompression stress resultant shifts from the centroid of the cross section to the ultimate compression block



Example 1—Tendon anchored externally.

resultant location. Finally, there is a third contribution from Fig. 2(c) to (d) generated from the increase in strand from the effective prestress level, f_{se}, to the ultimate strand strength f_{ps}. A simple equilibrium equation for the flexural strength capacity can be derived from these three component behaviors as

$$M_{n0} = -M_{bal} + P_{bal}e_C + A_{ps}(f_{ps} - f_{se})z \quad (3)$$

where M_{bal} is the balanced moment;

P_{bal} is the balanced axial force (resultant precompression);

e_C is the distance from the cross section centroid to the concrete compression block resultant (taken as positive if the resultant is below the centroid and negative if above the centroid);

and f_{se} is the effective stress of the prestressing strand (after stressing).

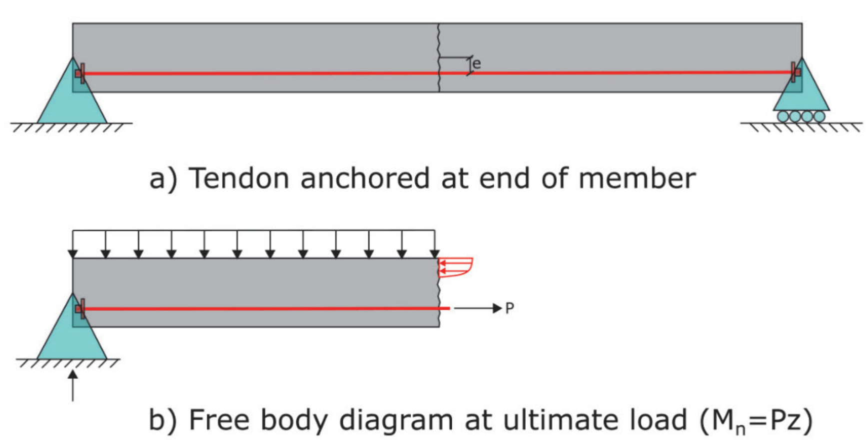
The balanced moment and axial force are calculated by theoretically removing the tendons from the member and replacing them with equivalent (balanced) loads, and then analyzing the structure for those loads. This equation is quite different from Eq. (1) for reinforced concrete and Eq. (2) for post-tensioned concrete. Equation (3), which is applicable to members without external net axial loads, can be compared directly with demand moments due to externally applied forces, M_u. Applying this equation to Example 1, M_{bal} and P_{bal} are zero because the tendon imparts no forces or prestrains on the member. There is also no increase in strand stress so $f_{ps} = f_{se}$ (based on the frictionless assumption). As such, Eq. (3) correctly calculates the flexural strength capacity of Example 1 as $M_n = 0$.

Now consider a second example identical to Example 1, but instead, with the strand anchored at the ends of the beam. In this case, the end anchorages introduce a compression P to the beam in addition to a balanced moment $-Pe$ caused by the strand eccentricity, where e is the depth between the centroid of the section and the prestressing strand. Although the strand itself again does not interact with the beam, the introduction of the beneficial compression strains by the anchorages generates considerable strength, which can be calculated using the first two terms of Eq. (3). Because there is no friction, the strand stress will not increase substantially due to loading, and it can be assumed that $f_{ps} = f_{se}$. The resulting flexural strength capacity of Example 2 can be calculated using Eq. (3), realizing that $-e_C$ is equivalent to $(z - e)$

$$M_n = -M_{bal} + P_{bal}e_C = Pe + P(z - e) = Pz$$

Lastly, consider a third example where instead of a strand being present at all, two hydraulic jacks apply a force P to the ends of the beam at the strand location from Example 2. It should be clear that, despite the lack of presence of any strands, this beam will have identical flexural strength capacity as Example 2, which can also be calculated using the first two terms of Eq. (3), with the jacking forces applied before the application of external load treated as a beneficial prestress force. In this case, $M_{bal} = -Pe$ and $P_{bal} = -P$, and the resulting capacity is $M_n = Pz$.

The previous three examples illustrate that the strength of prestressed members is derived primarily through beneficial prestrains and not by the presence of stressed strands in the cross section. This derivation also proves the concept of linear transformation presented by Lin and Burns.[3] This concept states that any transformation of cable profiles that does not result in any change in the calculated balanced loading also does not impact the resulting internal strength. If an interior support profile elevation is raised or



Example 2—Tendon anchored at beam ends.

lowered, and the corresponding midspan profiles are adjusted so that the resulting balanced load is unchanged, the resulting flexural strength capacity is also unchanged (if the change in tendon stress from f_{se} to f_{ps} is ignored). This further illustrates the independence between the flexural strength capacity and the location or presence of the strands.

RECOMMENDED SECONDARY FORCE APPROACH

This section will develop a secondary force approach that is both consistent with the new equilibrium approach and the intent of ACI 318, and is therefore suitable for use in design.

Equation (3) can be algebraically manipulated and expressed as

$$M_{n0} = A_{ps} f_{ps} z - M_{sec} + P_{sec} e_C \qquad (4)$$

where

$$M_{sec}\,(\text{secondary moment}) = M_{bal} - M_p \qquad (5)$$

$$P_{sec}\,(\text{secondary axial force}) = P_{bal} - P_p \qquad (6)$$

$$M_p\,(\text{primary moment}) = -A_{ps} f_{se} e$$

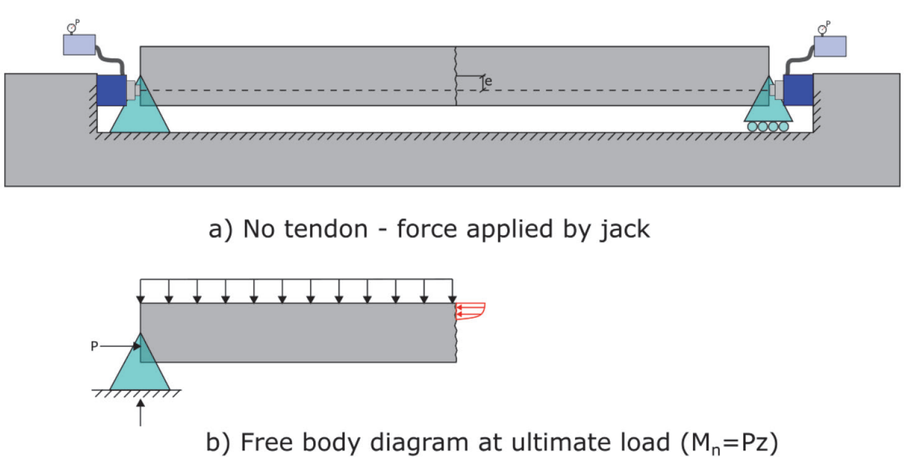
$$P_p\,(\text{primary axial force}) = -A_{ps} f_{se}$$

It should be apparent from Eq. (4) that a pseudo-capacity M_{n0}' calculated using Eq. (2), in addition to being modified by the secondary moment, must also be modified by the secondary axial force, P_{sec}. P_{sec} corrects for any difference between the primary strand force in the cross section and the actual precompression in the concrete generated from stressing the strands. Equation (2) is therefore only applicable when the resultant precompression in the cross section equals the primary axial force from the tendons. This allows a more general expression to be developed for the pseudo-capacity of post-tensioned sections, M_{n0}', including the effective stress of the strand

$$M_{n0}' = A_{ps} f_{ps} z + P_{sec} e_C \qquad (7)$$

Equation (7) should be used instead of Eq. (2) in the calculation of the pseudo-capacity of cross sections used with the secondary force approach. In cases where the resultant precompression in the cross section equals the primary axial force of the tendons, $P_{sec} = 0$ and Eq. (7) reduces to Eq. (2). Substituting Eq. (7) into (4) arrives at the fundamental relationship between the new equilibrium capacity M_{n0} and the pseudo-capacity M_{n0}' that is calculated with the secondary force approach. Equation (7) is compatible with ACI 318-19 Section 22.2.1.3 and is suitable for use in design.

$$M_{n0} = M_{n0}' - M_{sec} \qquad (8)$$



Example 3—Force P *applied by hydraulic jacks.*

CROSS SECTIONS WITH BOTH POST-TENSIONING AND NON-PRESTRESSED REINFORCEMENT

The previous derivations and examples have included only non-prestressed reinforcement or post-tensioning and have only been applicable when there are no net external axial loads. Most cross sections that are designed in post-tensioned structures contain both prestressing strands and non-prestressed reinforcement. Additionally, there is a tight interaction between flexural and axial demand and capacity, as explained in more detail in Appendix A. Equation (3) can be extended to consider both post-tensioning and non-prestressed reinforcement, along with any net external axial loads N as

Eq. (9) (new equilibrium approach)

$$M_n = -M_{bal} + \left(P_{bal} + N\right)e_C + \sum_{i=1}^{n,pt} A_{ps,i}\left(f_{ps,i} - f_{se,i}\right)z_i + \sum_{j=1}^{n,re} A_{s,j} f_{s,j} z_j \quad (9)$$

Eq. (7) can also be extended similarly as

Eq. (10) (secondary force approach)

$$M'_n = \sum_{i=1}^{n,pt} A_{ps,i} f_{ps,i} z_i + \sum_{j=1}^{n,re} A_{s,j} f_{s,j} z_j + \left(P_{sec} + N\right)e_C \quad (10)$$

When comparing these calculated flexural capacities to the demand flexure caused by external loads, ACI 318-19 requires that a strength reduction factor, ϕ, be applied to the calculated capacities. In typical calculations, this would be applied to M_n. Using the secondary effects approach, this is instead applied to the pseudo-capacity, M_n', while the secondary effects (M_{sec}) get applied a factor of 1.0. This small difference is not relevant to the overall behavior described in this document, and for this reason, the factor ϕ is not included in any of the equations and example calculations presented. However, it should be included as appropriate in real design calculations.

In determining the adequacy of a given design, the flexural capacity must be greater than or equal to the flexural demand.

$$M_u \le M_n \quad (11)$$

The relationship between Eq. (9) and (10) can be expressed similarly to Eq. (8) as

$$M_n = M_n' - M_{sec} \quad (12)$$

Substituting Eq. (12) into (11) and rearranging, the demand versus capacity equations can be expressed as

$$M_u + M_{sec} \le M_n' \quad (13)$$

$$M_n' = M_n + M_{sec} \quad (14)$$

Equation (13), used along with Eq. (10) for M_n' and Eq. (5) for M_{sec}, is the demand versus capacity relationship recommended for use with the secondary force approach for design. This provides a straightforward method to calculate secondary forces, which takes the ambiguity out of the ACI definitions for secondary forces, is consistent with the ACI definitions, and is generally applicable.

DIFFERENCE BETWEEN NEW EQUILIBRIUM AND RECOMMENDED SECONDARY FORCE APPROACH

The difference between the new equilibrium behavior and the recommended secondary effects approach is simply a numerical manipulation. It has been demonstrated that the real capacity behavior predicted by the new equilibrium equation is a function of the beneficial prestrain in the concrete generated by the act of stressing the strands. The new equilibrium approach can make it easier to intuitively understand the strength behavior in complex cases as it doesn't require any special considerations such as secondary effects. If the calculated flexural strength uses the entire ultimate stress in the strand, it is considered a pseudo-capacity that requires adjustments to get the correct demand versus capacity comparison. These adjustments, in the way of secondary forces, essentially move the secondary forces to the demand load side of the equation. The secondary force approach effectively offsets both the flexural capacity and the flexural demand by the secondary moment, as illustrated by Eq. (13) and (14). The physical meaning is simply the choice to apply the secondary effects along with the applied loads instead of in the calculation of the resistance at the time of stressing when they actually occur.

DESIGN EXAMPLES

Note: The reader is reminded that the strength reduction factor, φ, is not included in the following examples for the sake of clarity, although this factor should always be incorporated into real analysis and designs.

Examples 1 to 3 were originally calculated using the new equilibrium Eq. (3). Calculate the demand versus capacity ratios using the secondary effects approach (Eq. (7)).

Example 1—secondary force approach

Because this cross section contains a tendon but has no precompression, there is a secondary axial force that needs to be considered. The pseudo-capacities $M_n'^{+}$ and $M_n'^{-}$ can be calculated using Eq. (7) and using the same assumption in the original example that $f_{ps} = f_{se}$.

$$P_p = -P;\ P_{bal} = 0;\ P_{sec} = P_{bal} - P_p = P$$

$$M_n'^{+} = A_{ps} f_{se} z - P_{sec}(z - e) = Pz - Pz + Pe = Pe$$

$$M_n'^{-} = -A_{ps} f_{se} z + P_{sec}(z + e) = -Pz + Pz + Pe = Pe$$

This further illustrates why this term is called a pseudo-capacity as it is obvious that this member has no real capacity. However, applying this pseudo-capacity with the required secondary moment results in the correct demand versus capacity ratio.

$$M_{sec} = M_{bal} - M_p = 0 - (-Pe) = Pe$$

$$M_u + Pe \leq Pe \text{ (positive moment)}$$

$$M_u + Pe \geq Pe \text{ (negative moment)}$$

The only value of M_u that satisfies both the positive and negative moment equations is $M_u = 0$. As the new equilibrium equation does, the recommended secondary force approach also calculates the correct behavior that the application of any load will result in failure of the member.

Example 2—secondary force approach

Start by calculating the balanced, primary forces, and secondary axial force. Because the internal strand force is equal to the internal precompression resultant, the secondary axial force in this example is 0.

$$P_p = -P;\ P_{bal} = -P;\ P_{sec} = P_{bal} - P_p = 0$$

For comparison with the new equilibrium capacity calculated in Example 2, the positive moment capacity is calculated using Eq. (7).

$$M_n'^{+} = A_{ps} f_{se} z = Pz$$

$$M_{sec} = M_{bal} - M_p = -Pe - (-Pe) = 0$$

$$M_u \leq Pz \text{ (positive moment)}$$

which is identical to the result calculated using the new equilibrium equation (because the secondary effects are zero in this example).

Example 3—secondary force approach

This example does not have any tendons, so $M_p = P_p = 0$. The balanced forces are calculated from the loads applied by the hydraulic jacks

$$P_{bal} = -P;\ M_{bal} = -Pe;\ P_{sec} = P_{bal} - P_p = -P$$

This case has a secondary axial compression, which contributes to the flexural strength. For comparison with the new equilibrium capacity calculated in Example 3, the positive moment capacity is calculated using Eq. (7).

$$M_n'^{+} = -(P_{sec})(z - e) = Pz - Pe$$

$$M_{sec} = M_{bal} - M_p = -Pe$$

$$M_u - Pe \leq Pz - Pe$$

$$M_u \leq Pz \text{ (positive moment)}$$

As expressed earlier, both the demand and the capacity are offset by the secondary moment $-Pe$, which results in an identical demand versus capacity comparison as the new equilibrium calculation.

Example 4—practical design comparing both methods

The following example illustrates the flexural strength calculations at the cross section at the interior support and midspan of a 12 in. width of an 8 in. thick one-way slab, $f_c' =$ 5 ksi, a uniform distributed load of $w_u = 0.200$ klf (including self-weight), with a single one-half inch diameter unbonded monostrand with $f_{se} = 176.5$ ksi and f_{ps} assumed to be 195 ksi, and f_y of mild reinforcement of 60 ksi.

The flexural strength at cross section 4A using Eq. (9) is

$$M_n = -M_{bal} + P_{bal}e_C + A_{ps}(f_{ps} - f_{se})z + A_s f_y z$$

$$a = \frac{C}{0.85 \cdot b \cdot f_c'} = \frac{0.153 \cdot 195 + 0.2 \cdot 60}{0.85 \cdot 12 \cdot 5} = 0.82\,\text{in.}$$

$$z = -7 + (0.82/2) = -6.59\text{ in.}$$

$$M_n = -10.125 - 0.153 \cdot 176.5 \frac{\left(4 - \frac{0.82}{2}\right)}{12} +$$

$$0.153(195 - 176.5)\frac{-6.59}{12} + 0.2 \cdot 60 \cdot \frac{-6.59}{12} = -26.35\text{ ft} \cdot \text{kip}$$

The flexural strength at cross section 4A using Eq. (10) is

Because $P_{bal} = P_p$, $P_{sec} = 0$

$$M_n' = (A_{ps} f_{ps} + A_s f_y)z$$

$$M_n' = (0.153 \cdot 195 + 0.2 \cdot 60) \cdot (-6.59/12) = -22.97\text{ ft·kip}$$

$$M_{sec} = M_{bal} - M_p = 10.125 + 0.153 \cdot 176.5(-3/12) = 3.38\text{ ft·kip}$$

The results of the two methods can now be compared at section 4A

Method	M_u, ft·kip	M_n, ft·kip	Reserve capacity, ft·kip
Eq. (9) (new equilibrium)	–22.5	–26.35	3.85
Eq. (10) (secondary force)	–19.12	–22.97	3.85

It should be apparent that the two approaches are mathematically equivalent, so long as the secondary moments are included with the factored loads using the secondary force approach. Notice that, using Eq. (10), both the capacity and demand values are offset from the new equilibrium values by the magnitude of the secondary moment.

The flexural strength at cross section 4B using Eq. (9) is

$$M_n = -M_{bal} + P_{bal}e_C + A_{ps}(f_{ps} - f_{se})z + A_s f_y z$$

$$a = \frac{C}{0.85 \cdot b \cdot f_c'} = \frac{0.153 \cdot 195}{0.85 \cdot 12 \cdot 5} = 0.585 \text{ in.}$$

$$z = 7 - (0.585/2) = 6.71 \text{ in.}$$

$$M_n = 5.69 - 0.153 \cdot 176.5 \frac{\left(-4 + \frac{0.585}{2}\right)}{12} +$$

$$0.153(195 - 176.5)\frac{6.71}{12} = 15.62 \text{ ft} \cdot \text{kip}$$

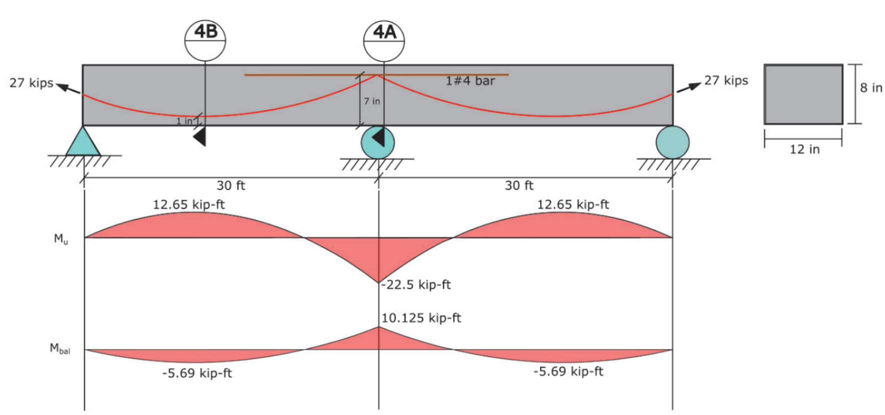


Fig. 4—Layout and moment diagram for Example 4.

The flexural strength at cross section 4B using Eq. (10) is

$$\text{Because } P_{bal} = P_p, P_{sec} = 0$$

$$M_n' = (A_{ps}f_{ps})z$$

$$M_n' = (0.153 \cdot 195) \cdot (6.71/12) = 16.68 \text{ ft·kip}$$

$$M_{sec} = M_{bal} - M_p = -5.69 + 0.153 \cdot 176.5(3/12) = 1.06 \text{ ft·kip}$$

The results of the two methods can now be compared at section 4B

Method	M_u, ft·kip	M_n, ft·kip	Reserve capacity, ft·kip
Eq. (9) (new equilibrium)	12.65	15.62	2.97
Eq. (10) (secondary force)	13.71	16.68	2.97

CONCLUSIONS

1. A new equation was derived using equilibrium to predict the strength capacity of post-tensioned sections. This capacity, M_n, is suitable for design and can be compared directly with factored demand loads, M_u, without any separate secondary force considerations.
2. The ACI 318 definitions for secondary forces in Sections 5.3.11 and 6.6.5.2 are too ambiguous and potentially confusing to apply correctly to modern analyses. For this reason, the ACI 318 definitions for secondary forces are not recommended for use in design. Instead, a definition for secondary forces was presented that is numerically consistent with the new equilibrium equation and is consistent with the ACI approach. In this approach, the ultimate stress in the strand f_{ps} is included in the cross section strain compatibility calculations. The pseudo-capacity that is calculated using this approach, M_n', must be used in conjunction with a demand that includes secondary force effects. This approach is suitable for design and usually more numerically convenient than the new equilibrium

equation because it is more consistent with reinforced concrete strength capacity calculations. The author recommends that the ACI definitions that pertain to secondary forces be changed to more numerically consistent definitions such as those recommended in this document. The new equilibrium equation, while not recommended for daily design, may be helpful to visualize and understand the strength behavior in some conditions.

3. The mathematical difference between the new equilibrium approach and the secondary force approach is merely that the demand and capacity are both offset by the secondary moment, as illustrated by Eq. (13) and (14).

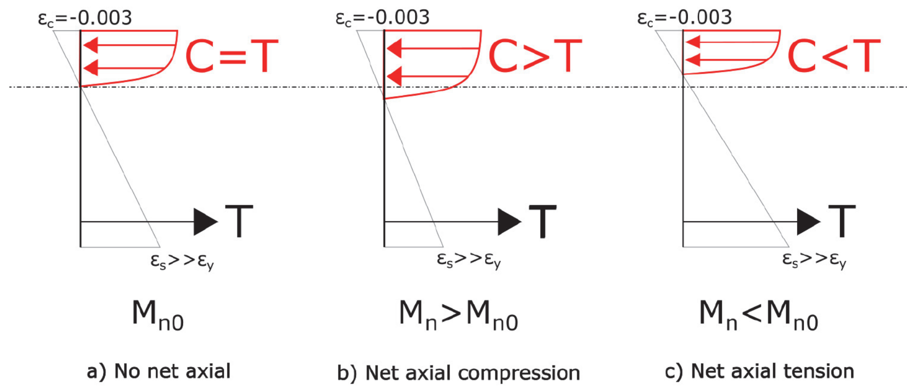


Fig. A1—Internal forces from strain compatibility analysis.

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APPENDIX A—INTERACTION BETWEEN FLEXURAL AND AXIAL FORCES

In a strain compatibility analysis, the compression strain is generally fixed and the tension strains are adjusted until the axial force resultant of the internal material stresses equals the externally applied demand axial force. The flexural capacity is then calculated from the integrated material stresses from the resulting strain condition. This illustrates the strong interaction between axial and flexural components that must be considered. Often in beams, the net axial force is zero, in which case the resultant compression force in the concrete equals the tension force in the reinforcement. However, if the demand forces include an external axial compression demand on the section, this must be considered in the equilibrium of the strain compatibility calculation; the compression block becomes larger, and the flexural capacity increases. Conversely, if there is an external axial tension demand on the section, the compression block becomes smaller and the flexural capacity decreases. This is illustrated in Fig. A1. The value of M_n with a net axial tension N is

$$M_n = A_s f_y z - N e_C \quad \text{(A1)}$$

where N is the net axial force demand on the section (tension positive, compression negative).

The second term in Eq. (A1) represents the flexural strength reduction caused by net tension, or the increase caused by net compression. Note that the lever arm may be slightly different due to the slightly different resultant concrete compression force depth.

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